the body and the virtual origin the observed wake converged, but at approximately 4 body-diameters downstream of the base the self-preservation or similarity region can be imagined to originate.

Unfortunately, it appears likely that the entire self-preservation and similarity concept does not represent closely what occurs in the wake in physical actuality. A recent paper by Ermshaus[4] gives very convincing proof that the supposed axially-symmetric wake-spreading behavior that would follow the $x^{1/3}$ law is not found to be confirmed by experiments, nor does the ratio remain constant that expresses the comparison between the maximum shear stress, represented by the quantity $\tau_m = -\rho(\overline{u'v'})_m$ (where u' and v' are the fluctuations of velocity in the x and r directions, respectively) to the square of the velocity defect at the middle of the wake. In the mixing-length theory used to derive the expressions given above there is implicit the hypothesis that $\tau_m/(U - U_\infty)_s{}^2$ must remain constant (where the s subscript denotes conditions at the centerline). The surprising result obtained by Ermshaus is that two-dimensional configurations (cylinders and narrow bands or beams) and axially symmetric configurations, as well, have to all intents and purposes the same exponent in the law describing the downstream spread of the wake. For all such configurations, to be precise, the width of the wake appears to grow approximately according to $x^{0.44} = x^{1/2.25}$, not the $x^{1/2}$ law predicted on basis of similarity theory for two-dimensional bodies, nor according to the $x^{1/3}$ law, predicted according to classic theory, for axially symmetric bodies. Consequently, even though Eqs. (6) and (7) appear to be well-substantiated by the faired data-plots given by Chevray, some caution should be exercised in trying to apply to quite different situations these particular results that have come from one kind of body shape and for which the asymptotic state apparently was not yet attained (if it ever would be).

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Reply by Authors to R. H. Cramer

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IN the case of axially-symmetric turbulent wake flow, the following equations for the velocity profile and the wake width were formulated simply from available information:

and

$$(U_\infty - U)/U_\infty = [x^2/(C_D d^2)]^{-1/3} f(\eta)$$

$$b = B(C_D d x)^{1/3} \tag{1}$$

where

$$\eta = y/b$$

These equations were then used to obtain the solution to the governing differential equations. The basis for formulating Eq. (1) is that information (Ref. 1, pp. 686, 691–2) applicable to two-dimensional wake flows, viz.,

$$(U_\infty - U)/U_\infty = [x/(C_D d)]^{-1/2} f(\eta)$$

$$b = B(C_D d x)^{1/2}$$

whereas for axially symmetric wake flow;

and

$$(U_\infty - U)/U_\infty \sim x^{-2/3}$$

$$b \sim x^{1/3}$$

Thus, by considering the dimensions of the velocity profile and the wake width, Eq. (1) was formulated.

L. M. Swain, in her pioneering 1929 paper,[2] elaborately worked out a similar form of Eq. (1) by using an order-of-magnitude analysis. At that time apparently no such information was available.

The boundary-layer equations, which are applicable for the wake (Ref. 1, p. 682) has been applied to obtain a solution. Because such a boundary-layer equation has been derived under the assumption of thicknesses that are small with respect to x, we obtain a solution which we consider more applicable to the far wake.

Swain considered, in addition, a second term involving $x^{-7/3}$ which is more significant in the near wake region as the body is approached.[2]

References

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Comment on "Added Mass of a Circular Cylinder in Contact with a Rigid Boundary"

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THE added mass of a circle tangent to a ground plane that is determined in the issue of January 1972,[1] was first calculated by Taylor[2] over forty years ago. Taylor actually calculated the added masses of a family of lenses consisting of two circular arcs. The family passes from a flat plate through a single circle to a pair of tangent circles. This last limiting case may be regarded as a circle tangent to a ground plane. In addition to this family, Taylor also considered other two-dimensional sections having a line of symmetry. Among these are a section whose upper half is defined by a symmetric parabolic arc, one whose upper half is in the shape of an isosceles triangle and one whose upper half is defined by a pair of semicircles.

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